USING REAL OPTION ANALYSIS TO IMPROVE CAPITAL BUDGETING DECISIONS WHEN PROJECT CASH FLOWS ARE SUBJECT TO CAPACITY CONSTRAINTS

ABSTRACT

When a capacity constraint exists, using net present value analysis to make capital budgeting decisions risks improperly estimating expected cash flows. The may lead to decision errors due to incorrect valuation. Using real option analysis for those cash flows that are subject to capacity constraints may improve valuation estimates. This requires the analyst to identify the implicit option created by the capacity constraint, and determine values for the underlying variables that affect the value of the real option. These variables include the current value and volatility of the subject matter of the option (unlike the valuation of financial options, this value will not typically be a market price), and the "strike price", the level at which the constraint applies. This paper examines the valuation problem presented by a capacity constraint and illustrates how real option valuation can improve a capital budgeting analysis.

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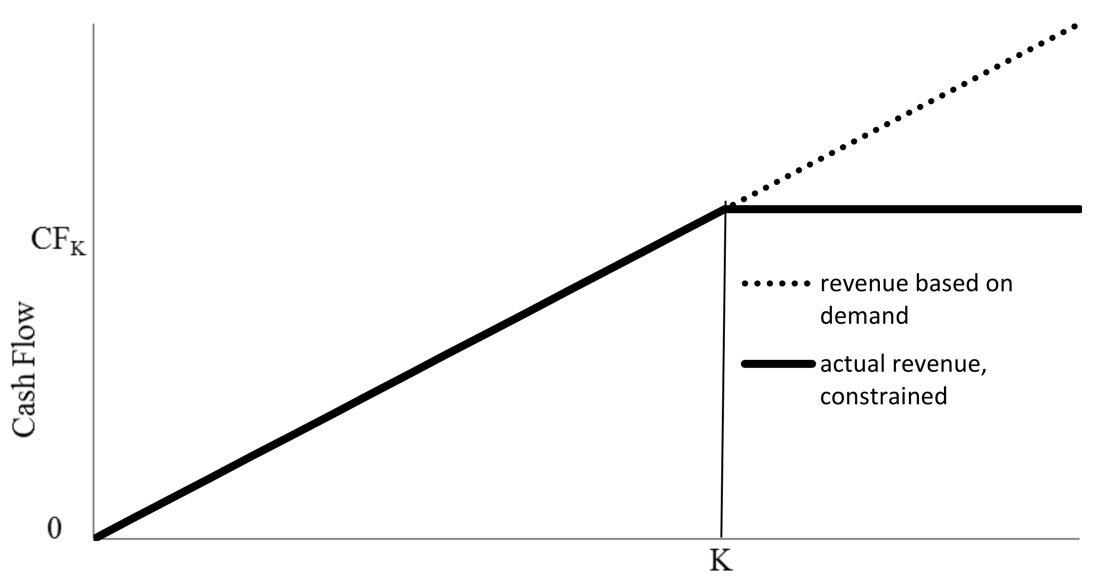
INTRODUCTION

Net present value analysis requires a financial manager to forecast expected net cash flows and discount them using an appropriate required return. Some projects have cash flows that are limited by capacity constraints. These constraints may also interfere with obtaining estimates of expected cash flows. For example, a real estate developer evaluating the feasibility of developing a hotel in a particular market may have accurate information about occupancy rates for similar properties already located in that market. However, on those days that the existing properties all operate at capacity, it is not possible to observe actual demand. Basing the valuation on an assumption that the proposed project can capture a portion of observed demand will underestimate the actual value of the project. Even when an analyst has good information about total demand, if the nature of the project (e.g., the size of a facility or the nature of its production process) create a limit on the revenue that can be realized in any particular period, a valuation that relies on demand without considering the effect of the constraint may overestimate project value. Figure 1 illustrates how a capacity constraint limits the ability to observe and/or to generate revenue from actual demand. The demand appears on the x-axis and the cash flow associated with the demand on the y-axis. When there is a capacity constraint at K, demand exceeding that level appears as demand of K units and revenue that can be generated is capped at CF_K.

Figure 1. Cash flow subject to a maximum constraint, K.



One way to include the effect of a capacity constraint when valuing a project affected by the constraint is to use real option analysis. The effect of the capacity constraint illustrated in

Figure 1 has the same pattern as the payoff profile for an option. This suggests that option pricing principles may be useful provided the option can be identified and appropriate values determined for those variables needed to value the real option.

APPLICATIONS OF REAL OPTION VALUATION

The real option literature suggests that real options analysis may be more accurate than net present value for: mineral production projects (Davis, 1996; Mann, Goobie and MacMillan, 1992; Sick, 1990; Palm, Pearson and Read, 1986; and Brennan and Schwartz, 1985); real estate development (Rocha, Salles, Alcaraz Garcia, Sardinha and Teixeira, 2007; Williams, 1991; and Titman, 1985); and mergers and consolidations (Lambrecht, 2004; and Smit, 2001). The real option characteristics examined in connection with these projects do not consider the effect of capacity constraints.

In other application of real option analysis, even when the project can be appropriately valued using net present value principles, some changes in the business environment create fundamental changes in a business that can best be valued using real option analysis. Trigeorgis, 1993, shows how option pricing improves valuation from net present value alone when a project can be expanded in response to greater than expected demand. McDonald and Siegel, 1985, and Brennan and Schwartz, 1985, demonstrate use of option analysis to value an option to shut down. Other research uses real option analysis to value the option to abandon (Myers and Majd, 1990) or to wait and begin the project at a later date (Quigg, 1993). Real option analysis has also been used to determine the optimal initial investment when there may be value to expanding or reversing an investment in response to changes in demand (Abel, Dixit, Eberly and Pindyck, 1996). See also Bøckman, Fleten, Juliussen, Langhammer, and Revdal, 2007.

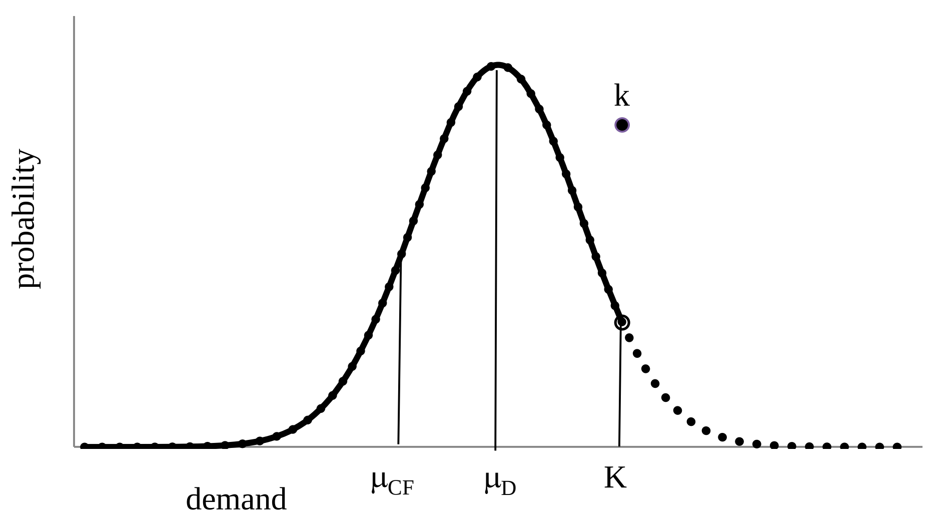
These foregoing studies consider the effect of a single future event that fundamentally alters future project cash flows and hence the project's value. This is analogous to the payoff on a financial option depending on whether it is in- or out-of-the-money based on the market price at a future date. Additionally, real option analysis is useful when a projects' periodic cash flows have option characteristics. This analysis requires the financial manager to separately value the option associated with each cash flow and include the aggregate value of all such options in the overall project value. Briys, Crouhy and Schöbel, 1991, use this approach to value interest rate caps, floors and collars, multi-period financial contracts. It has also been used to value projects with flexibility in product mix or in production methods. See, e.g., Gengtsson and Olhager, 2002; Andreou, 1990; Triantis and Hodder, 1990; Kulatilaka, 1988 and 1993; and Margrabe, 1978.

REAL OPTION VALUTION FOR CONSTRAINED CASH FLOWS

This paper examines the use of real option analysis for projects with cash flows that are subject to a capacity constraint. For these projects, cash flows increase or decrease with demand until reaching the constraint, at which time additional increases or decreases in demand have no effect on cash flow. The upper or lower limit associated with a capacity constraint creates option characteristics in the cash flows. Real option analysis may provide a more accurate measure of project value than traditional net present value analysis. The paper explains how to disaggregate the capacity constrained cash flow in order to use real option analysis and then describes the methodology of real option valuation.

When there is a capacity constraint, increases or decreases in demand lead to higher or lower revenue. With a constraint on production capacity, when demand exceeds the maximum capacity, cash flows no longer reflect demand but instead reflect the constraint. Observing cash flows. Figure 2 illustrates the how the observed demand may differ from actual demand when there is a capacity constraint at ***K***. The dotted line shows the distribution of actual demand. The solid line together with the point "***k***" is the distribution of observed demand based on cash flows. The aggregate probability of demand greater than or equal to K is the probability associated with the point ***k***. As a result, the mean demand based on observed cash flows, μ_{CF}, falls to the left of the actual mean for demand, μ_D. Present value estimates using mean cash flow underestimate those based on mean demand. Since actual demand is not observed, even if a potential new entrant into a market is able to accurately estimate the portion of demand it will be able to capture, it may not be able to accurately estimate currently unmet demand.

Figure 2. Distribution of actual and constrained demand



If a new entrant faces a capacity constraint similar to existing firms in the market, even if could estimate the mean of actual demand accurately, estimating cash flows based on actual demand will lead to an overestimate actual value, since in some instances the new entrant's own capacity constraint will limit its income when demand is high. Deviations of actual demand from mean demand on cash flow have an asymmetrical effect on cash flow. When demand is less than the mean, cash flows decline, but when demand exceeds the mean, cash flow increases are capped due to the capacity constraint. Valuation using mean demand overestimates cash flows. The magnitude of the error will depend on volatility of demand.

Valuing the project using real option analysis rather than traditional net present value overcomes this problem. A cash flow subject to a capacity constraint is first decomposed into an unconstrained cash flow and an option-like cash flow. This real option will have a value of zero over for a portion of the demand range and a value that is linearly related to the demand over the rest of the range. The analyst then estimates value for both the unconstrained cash flow and the real option. Combining these two values provides an estimate for the value of the constrained. This calculation must be done for each project period that is affected by the capacity constraint.

So, for example, if the constraint affects the maximum cash flow that can be realized in a day, cash flows are comprise of a series of options that expire daily. It is necessary to value daily cash flows in order to accurately value the constrained cash flows.

To contrast net present value analysis with real option analysis, consider a project which has revenue that is subject to a capacity constraint, such as that illustrated in Figure 1. Net present value analysis aggregates the present value of all future expected revenue for the project. That is,

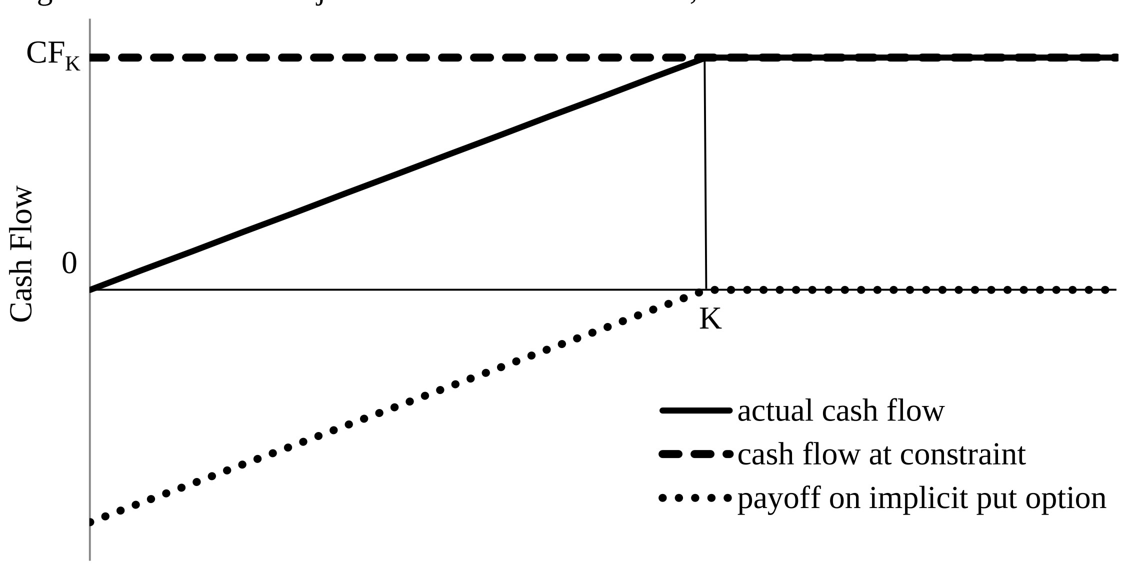
$$Present\ value\ of\ revenue = \sum_{t=1}^{T} \frac{E(Revenue_t)}{(1+r_{Rev})^t}. \tag{1}$$

Using real option analysis, each period's revenue is disaggregated into an unconstrained cash flow and a real option. Figure 3 illustrates this separation. The heavy solid line indicates the actual cash flow as a function of demand. The dashed line equal to $\boldsymbol{CF_K}$ is the unconstrained payment, independent of demand. The dotted line is the real option. It has the same payoff as a written put option with demand as the underlying asset and a strike price of $\boldsymbol{K}$. Combining the unconstrained cash flow and the real option gives the same revenue as the solid line. So the combined value of the unconstrained cash flow and the put option is the value of the revenue,

$$Present\ value\ of\ revenue = \sum_{t=1}^{T} \left(\frac{Revenue_K}{\left(1+r_f\right)^t} - P_t \right), \tag{2}$$

where: $\boldsymbol{Revenue_K}$ is the revenue when demand is at capacity, $\boldsymbol{K}$; $\boldsymbol{r_f}$ is the risk free rate; and $\boldsymbol{P_t}$ is the value of the real option the expires at time $\boldsymbol{t}$. The risk free rate is the used to discount revenues because the revenue is known. Aggregating each period's values provides the alternative valuation that specifically accounts for the option characteristics of project cash flows.

Figure 3. Cash flow subject to a maximum constraint, K.



The value of a project subject to a capacity constraint is thus equal to the value of the project if it produces at capacity each period (with revenue discounted at the risk free rate since there is no variance in revenue) less the aggregate value of the implicit put options. These options account for the reduced value due to production at less than capacity.

Valuing each put option is straightforward. Assuming the distribution of demand is loglinear, the real option is for valuation purposes equivalent to a put option on a commodity with a strike price equal to the cash flow at expected demand, which for valuation purposes is a commodity futures' price. From Black (1976), the value of the put equals:

$$P_t = \frac{1}{(1+r)^t}\left(Revenue_K N(-d_2) - Revenue_{Dt} N(-d_1)\right), \tag{3}$$

where:

$$d_1 = \frac{ln\left(\frac{Revenue_{Dt}}{Revenue_K}\right)}{\sigma t^{\frac{1}{2}}} + \frac{\sigma t^{\frac{1}{2}}}{2},$$

$\boldsymbol{d_2=d_1-\sigma t^{1/2}}$ and $\boldsymbol{N(-d_i)}$ is the value of the standard normal cumulative distribution function evaluated at $\boldsymbol{-d_i}$. $\boldsymbol{Revenue_{Dt}}$, in this real option a value analogous to the market price of the underlying asset for a traditional option, is the revenue based on time-t actual demand; $\boldsymbol{Revenue_K}$, the "strike price" for the real option, is the revenue at capacity. Other variables have the same meaning as in traditional options; **r** is the risk free interest rate, **t**, the time to expiration of the option is the period for which the value of the cash flow is being calculated, and **σ** is the standard deviation of demand, the underlying asset. The mean of actual demand and the standard deviation of demand are derived from observed demand using censored data moment estimation methods. See e.g. Tiku (1967).)

Replacing $\boldsymbol{P_t}$ in the expression in the parentheses in Eq. 2 with the Eq. 3 and rearranging indicates that the time **t** cash flow on a capacity constrained projected can be alternatively expressed as:

$$Present\ value\ of\ revenue = \sum_{t=1}^{T}\left(\frac{Revenue_K}{\left(1+r_f\right)^t}N(d_2) + \frac{Revenue_{Dt}}{\left(1+r_f\right)^t}\left(1-N(d_1)\right)\right) \tag{2'}$$

The initial term on the right-hand side of equation 2' is the value of a "cash-or-nothing" option that pays the present value of the cash flow at capacity if demand is greater than or equal to the "strike price", i.e., capacity constraint. The second term is the present value of cash at the mean actual demand less the value of a "share-or-nothing" option with a strike price equal to cash flow at the capacity constraint.

Figure 4. Cash flow subject to a minimum constraint, K.

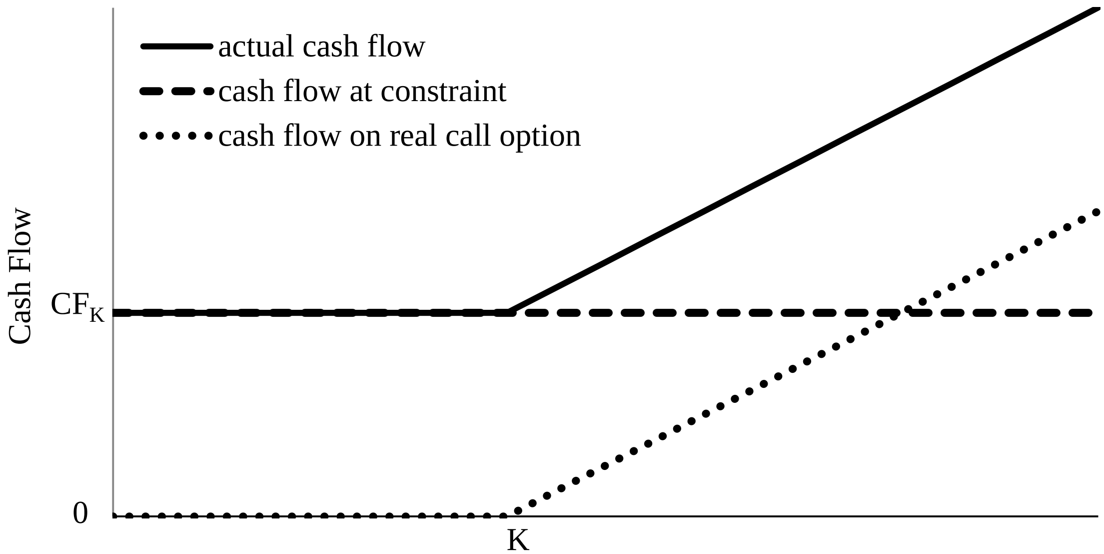


Figure 4 illustrates cash flow subject to a minimum constraint (the solid line in that Figure). Decomposition of this cash flow to facilitate real option analysis uses a fixed periodic cash flow at the minimum capacity level (the dashed line), together with a call option having a "strike price" equal to the capacity constraint (the dotted line). The real option equation to value this cash flow is:

$$Present\ value\ of\ revenue = \sum_{t=1}^{T}\left(\frac{Revenue_K}{\left(1+r_f\right)^t} - C_t\right), \tag{4}$$

where $\boldsymbol{C_t}$, is the value of a call option that expires at time $\boldsymbol{t}$ with a strike price equal to the revenue at the minimum capacity and value of time $\boldsymbol{t}$ cash flow in excess of the minimum capacity equals:

$$C_t = \frac{1}{(1+r)^t}\left(Revenue_{Dt}N(d_1) - Revenue_K N(d_2)\right). \tag{5}$$

Variables are defined as in Equation 3.

CONCLUSION

Real option analysis allows valuation of projects with capacity constrained cash flows. This method expressly incorporates the effect of the non-linearity of cash flows due to a capacity constraint. Because the cash flow includes the effect of the option characteristics, the value obtained using this method is more accurate than basing value on net present value of expected cash flow. In addition, because the risk free rate is used for valuing the option, it is not necessary to obtain a risky-cash-flow required return in order to value the constrained cash flows. However, that will still be necessary in valuing other project cash flows that are not subject to capacity constraints.

The valuation method described in this paper is consistent with the one proposed by Deng, Johnson and Sogomonian, 2001, for valuing peak load electrical production. "Peak load production" is characterized by zero production until high levels of power demand cause price to increase to the level at which producing is economical. Deng, et al., value time-t production using a "spark spread call option", where the payoff on the option depends on the spread between time-t price at which electricity can be sold and the production cost. Extending their analysis as described herein allows valuation not just of the time-t production decision but of the peak load producer itself by aggregating the time-t "spark spread option" values for all production periods. Since many different types of production and service business face capacity constraints of one kind or another, it is expected that this valuation technique will have wide application.

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